

# TRANSFORMATIONS 4024

Compiled by Mustafa Asif

## 39. Transformations

- use the following transformations of the plane: reflection (M), rotation (R), translation (T), enlargement (E) and their combinations
- identify and give precise descriptions of transformations connecting given figures
- describe transformations using coordinates and matrices

If  $M(a) = b$  and  $R(b) = c$  the notation  $RM(a) = c$  will be used.

Invariants under these transformations may be assumed.

Singular matrices are excluded.

Transformations – references removed to shear and stretching.

Notes by Sarwar Khan

[https://drive.google.com/open?id=1kOOY781PhLBtHHaLzBdNOV1ozqC\\_jYCB](https://drive.google.com/open?id=1kOOY781PhLBtHHaLzBdNOV1ozqC_jYCB)

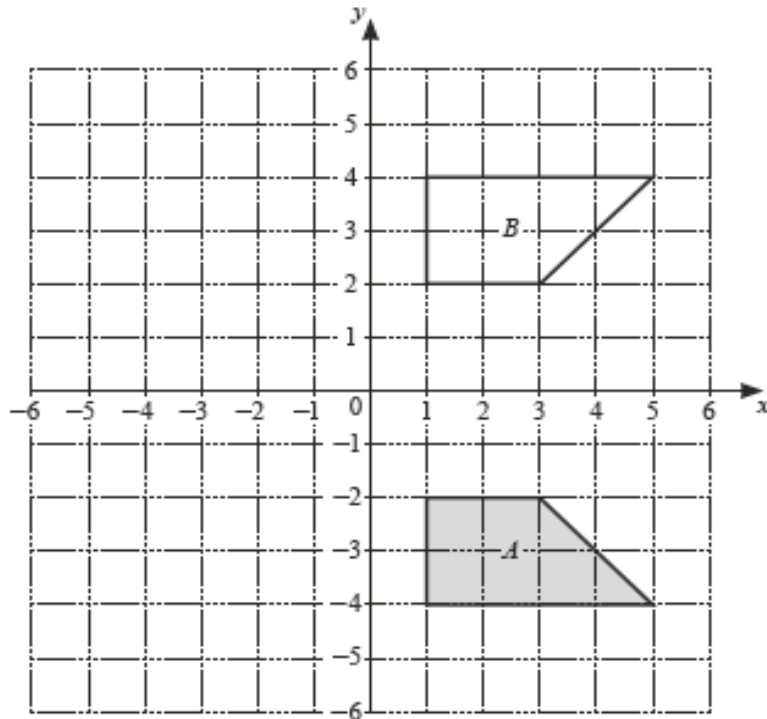
Transformation playlist(very helpful for understanding basics)

<https://www.youtube.com/watch?v=P1V0o7BxShk&list=PLSunMkRHiawC2PvT03UDLjL2Mcldorz7C>

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1

M/J19/11/21



Shapes *A* and *B* are drawn on the grid.

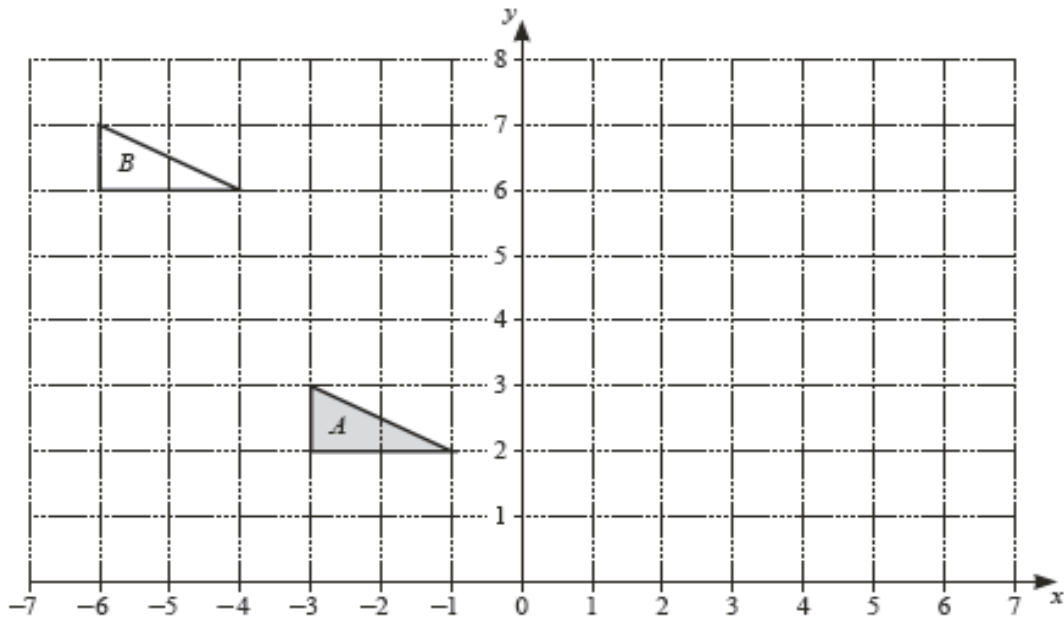
- (a) Draw the image of shape *A* after a translation with vector  $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ . [1]
- (b) Draw the image of shape *A* after an enlargement with scale factor  $-\frac{1}{2}$  and centre  $(1, 0)$ . [2]
- (c) Shape *A* is mapped onto shape *B* by the **single** transformation P.
- (i) Describe fully the transformation P.  
 ..... [2]
- (ii) Find the matrix representing transformation P.

$\left( \quad \right)$  [1]

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2

M/J19/12/16



Triangle  $A$  and triangle  $B$  are drawn on the grid.

- (a) Describe fully the single transformation that maps triangle  $A$  onto triangle  $B$ .

.....  
..... [2]

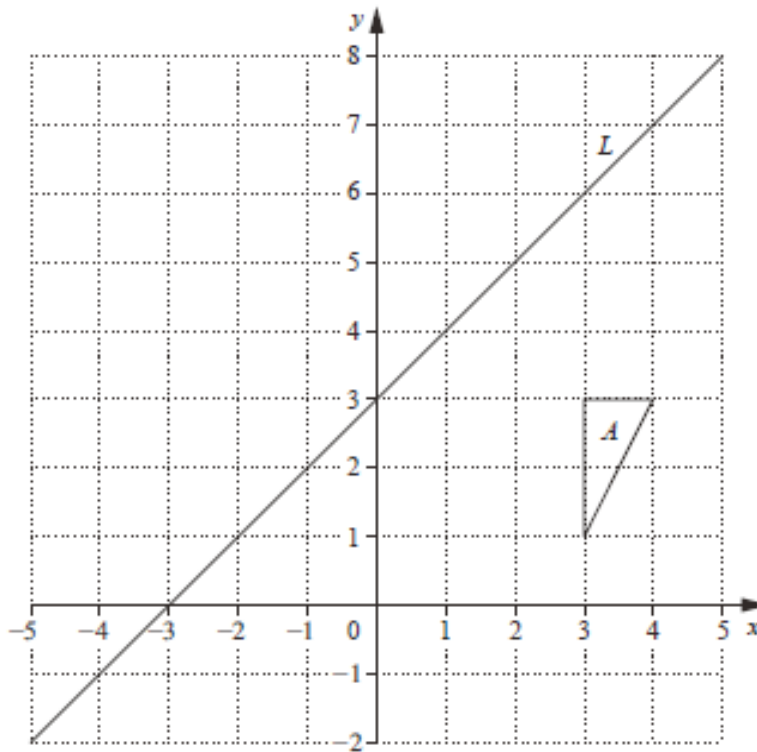
- (b) Triangle  $A$  is mapped onto triangle  $C$  by an enlargement with centre  $(0, 3)$  and scale factor  $-2$ .

On the grid, draw triangle  $C$ . [2]

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SP18/02/3



The grid shows triangle  $A$  and line  $L$ .

- (a) (i) Triangle  $A$  is mapped onto triangle  $B$  by a reflection in line  $L$ .

Draw and label triangle  $B$  on the grid.

[2]

- (ii) Triangle  $A$  is mapped onto triangle  $C$  by a clockwise rotation of  $90^\circ$ , centre  $(0, 3)$ .

Draw and label triangle  $C$  on the grid.

[2]

- (iii) Triangle  $C$  is mapped onto triangle  $D$  by a reflection in line  $L$ .

Describe the single transformation that maps triangle  $B$  onto triangle  $D$ .

Answer .....

..... [3]

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- (b) The line  $P$  is parallel to line  $L$  and passes through the point  $(0, 5)$ .

Find the equation of line  $P$ .

*Answer* ..... [2]

- (c) The line  $R$  is perpendicular to line  $L$  and passes through the origin  $(0, 0)$ .

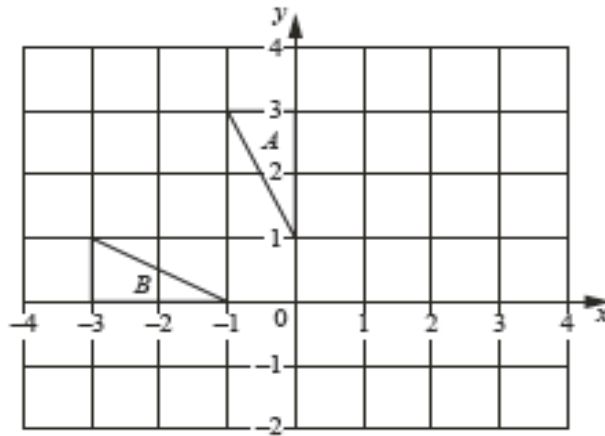
Find the equation of line  $R$ .

*Answer* ..... [1]

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O/N18/11/22



The diagram shows triangles  $A$  and  $B$ .

- (a) Describe fully the single transformation that maps triangle  $A$  onto triangle  $B$ .

.....  
..... [2]

- (b) Triangle  $A$  is mapped onto triangle  $C$  by a rotation, through  $90^\circ$  clockwise, centre  $(0, 0)$ .

Draw, and label, triangle  $C$  on the diagram. [2]

- (c) Triangle  $B$  is mapped onto triangle  $C$  by the transformation  $T$ .

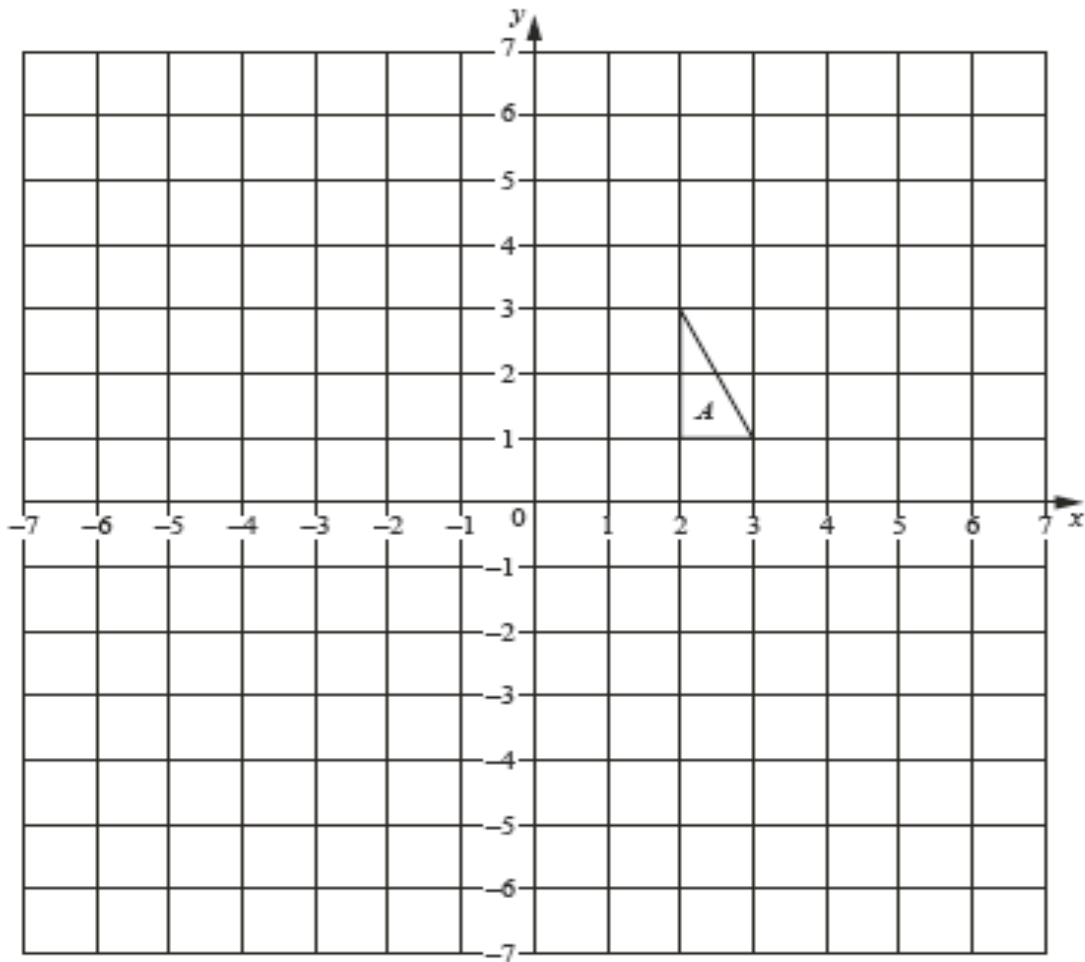
Find the matrix that represents the transformation  $T$ .

Answer  $\left( \begin{array}{cc} & \\ & \end{array} \right)$  [1]

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O/N18/22/7



Triangle  $A$  is drawn on the grid.

(a) Transformation  $P$  is represented by the matrix  $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ .

$P$  maps triangle  $A$  onto triangle  $B$ .

(i) Draw and label triangle  $B$ .

[2]

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(ii) Describe fully the single transformation P.

.....  
..... [2]

(iii) Write down the ratio area of triangle A : area of triangle B.

Answer ..... : ..... [1]

(b) Transformation Q is represented by the matrix  $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ .

Q maps triangle B onto triangle C.

Draw and label triangle C. [2]

(c) Transformation Y is represented by the matrix  $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ .

Y maps triangle A onto triangle D.

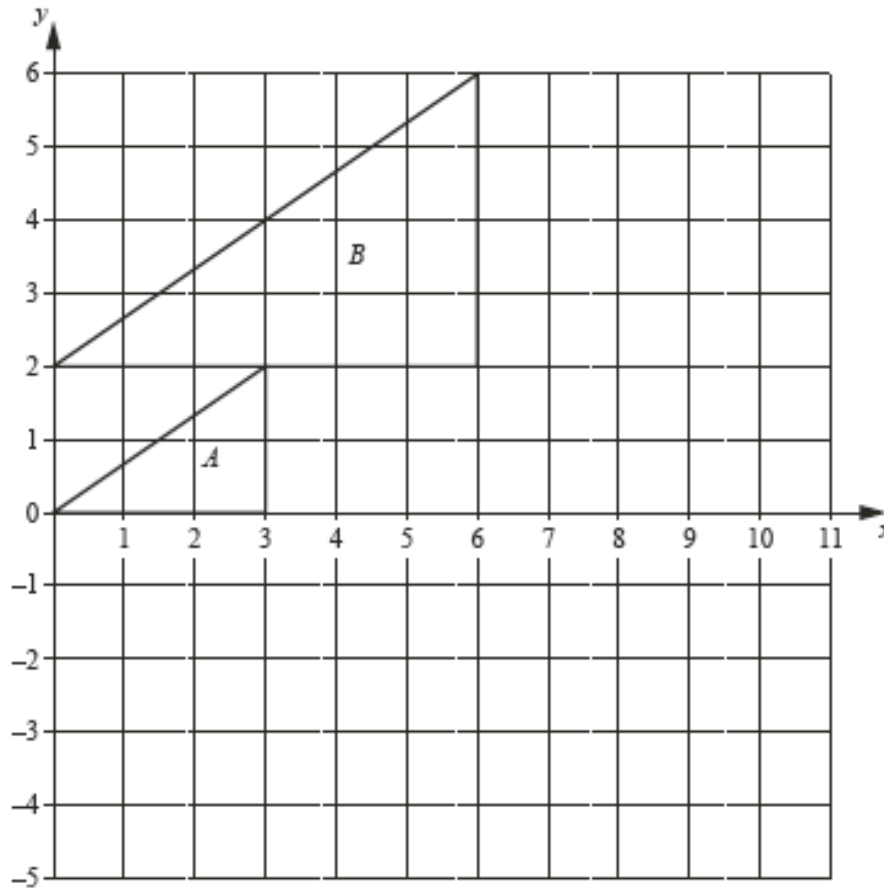
Find the matrix that represents the transformation that maps triangle D onto triangle A.

Answer  $\begin{pmatrix} & \\ & \end{pmatrix}$  [2]



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Triangle *A* is mapped onto triangle *B* by a translation, **followed by** an enlargement with centre  $(10, -4)$ .  
 The translation maps triangle *A* onto triangle *C*.  
 The enlargement maps triangle *C* onto triangle *B*.

(a) Write down the scale factor of the enlargement.

*Answer* ..... [1]

(b) Draw triangle *C* on the grid.

[2]

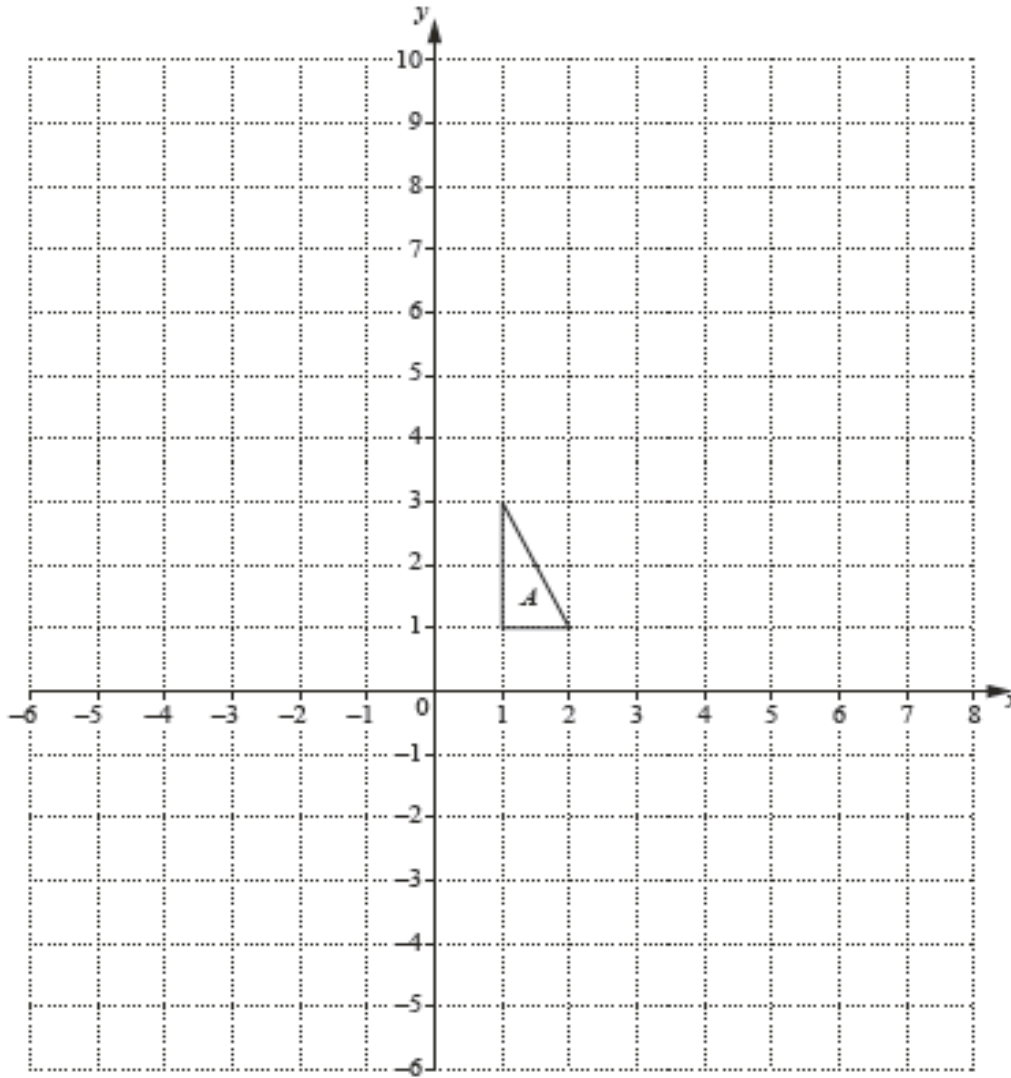
(c) Find the column vector that represents the translation that maps triangle *A* onto triangle *C*.

*Answer*  $\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$  [1]

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7 (a) Triangle  $A$  is shown on the grid.

O/N17/21/10(a)



(i) Triangle  $A$  is mapped onto triangle  $B$  by a rotation of  $180^\circ$  about point  $(2, -1)$ .

Draw and label triangle  $B$  on the grid.

[2]

(ii) Triangle  $A$  is mapped onto triangle  $C$  by the transformation represented by the matrix  $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ .

Draw and label triangle  $C$  on the grid.

[2]

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(iii) Write down the matrix that represents the transformation that maps triangle  $C$  onto triangle  $A$ .

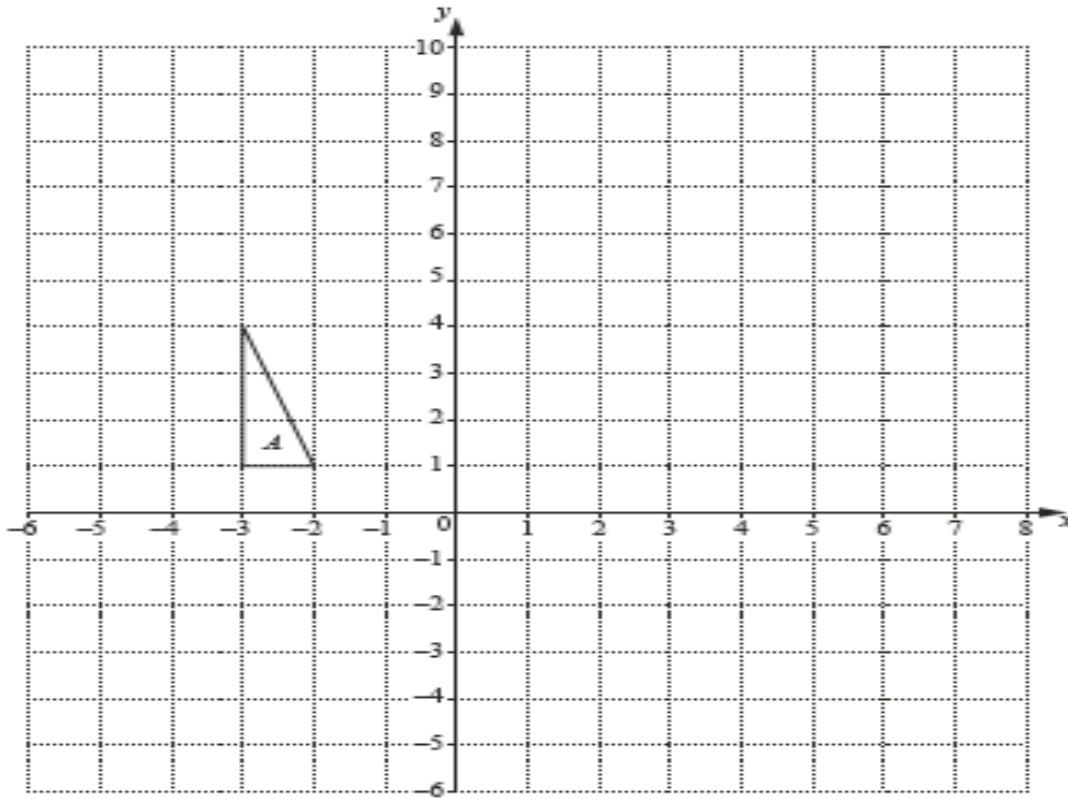
Answer  $\begin{pmatrix} & \\ & \end{pmatrix}$  [1]

(iv) Describe fully the single transformation that maps triangle  $C$  onto triangle  $B$ .

.....  
..... [3]

8 (a) Triangle  $A$  is shown on the grid.

O/N17/22/4(a)



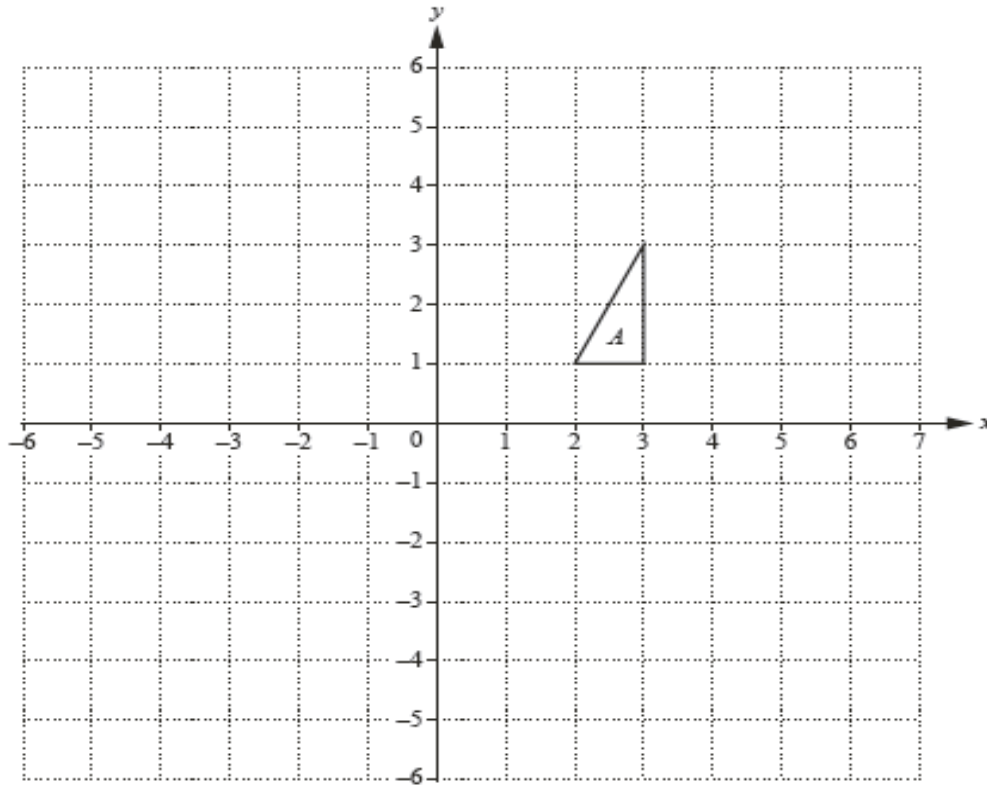
(i) Triangle  $A$  is mapped onto triangle  $B$  by a translation of  $\begin{pmatrix} 7 \\ -5 \end{pmatrix}$ .  
Draw and label triangle  $B$  on the grid. [2]

(ii) Triangle  $A$  is mapped onto triangle  $C$  by an enlargement scale factor  $-2$ , centre  $(-1, 2)$ .  
Draw and label triangle  $C$  on the grid. [2]

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9 The diagram shows triangle  $A$ .

M/J17/11/17(a)



(a) Triangle  $B$  is the image of triangle  $A$  after reflection in the line  $y = -1$ .

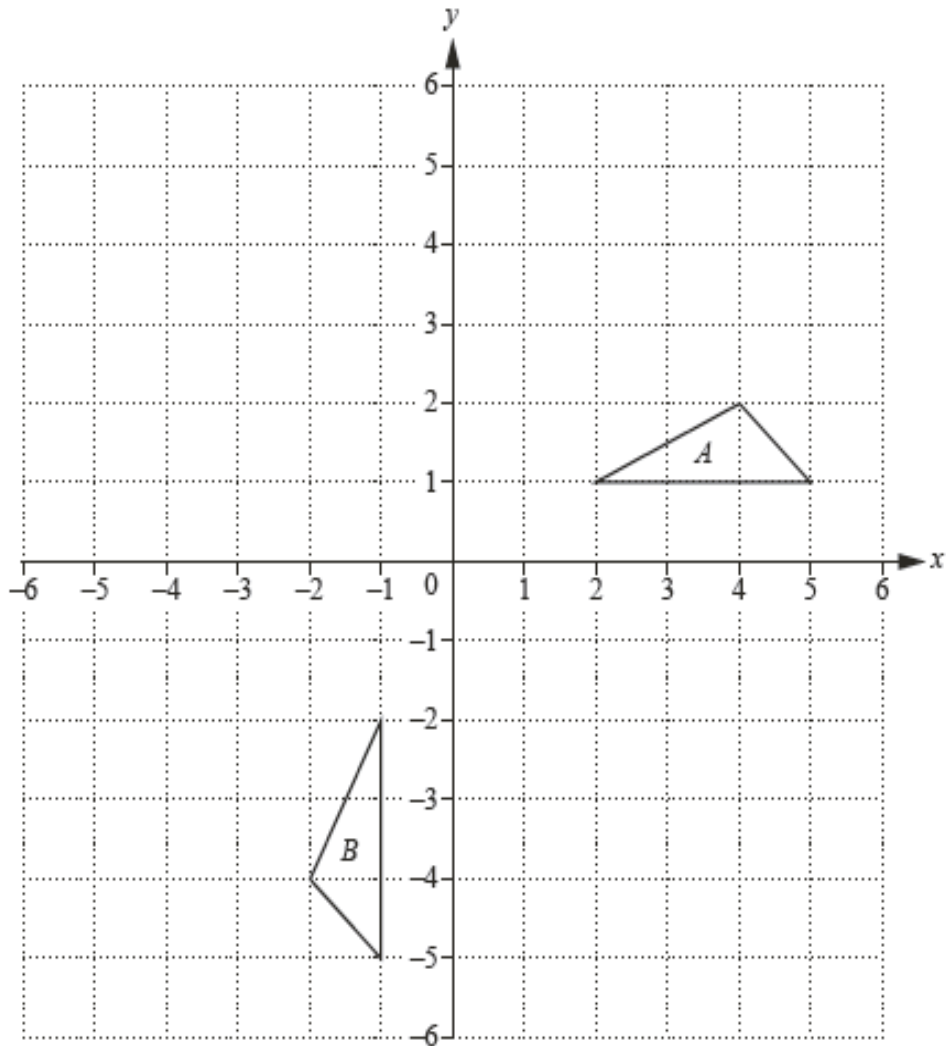
Draw and label triangle  $B$  on the diagram.

[1]

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10 The diagram shows triangles  $A$  and  $B$ .

M/J17/12/15



(a) Describe fully the single transformation that maps triangle  $A$  onto triangle  $B$ .

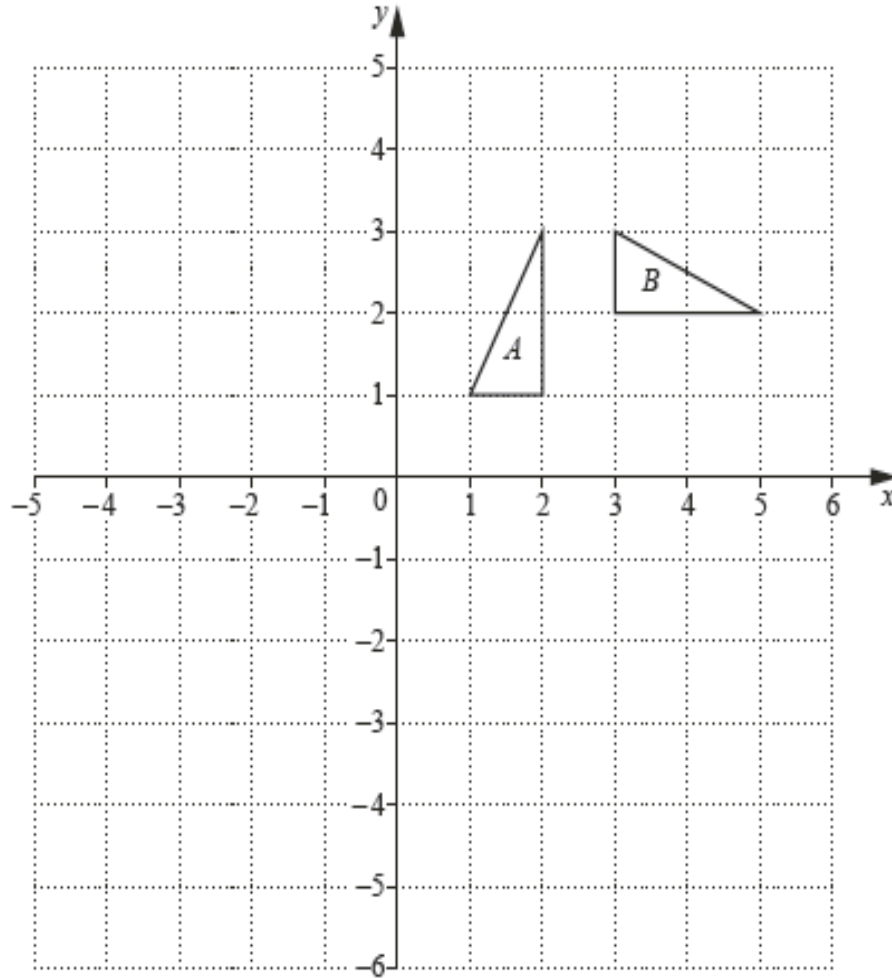
Answer .....

..... [2]

(b) Triangle  $A$  is mapped onto triangle  $C$  by a rotation,  $90^\circ$  anti-clockwise about the origin.

On the diagram, draw triangle  $C$ . [2]

11



- (a) Describe the single transformation that maps triangle  $A$  onto triangle  $B$ .

Answer .....

..... [2]

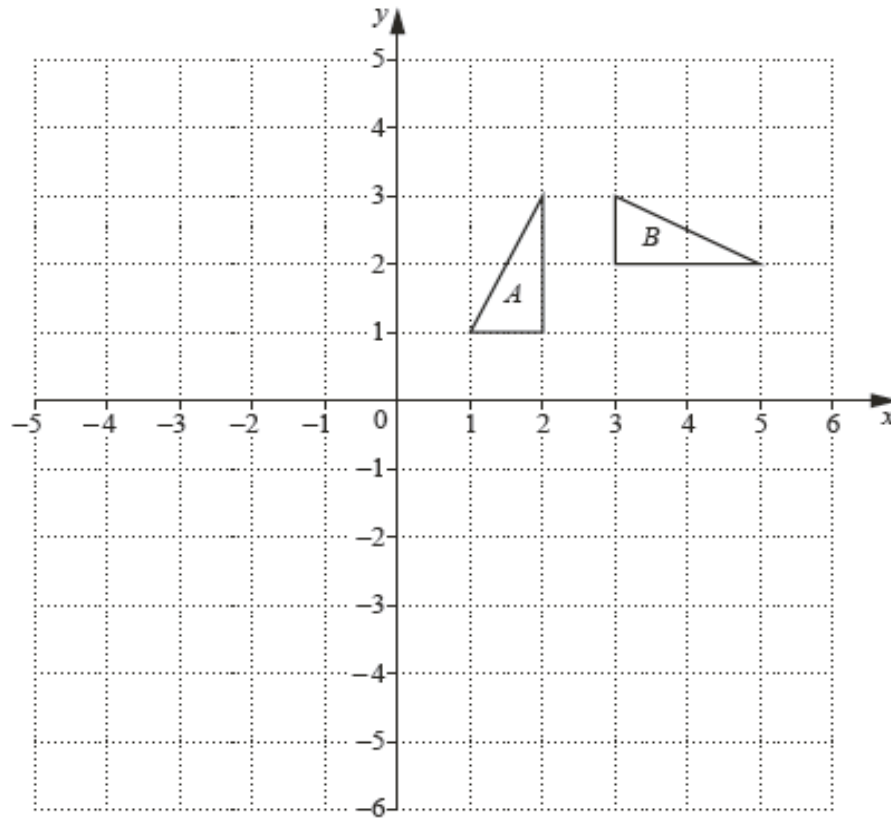
- (b) Triangle  $A$  is mapped onto triangle  $C$  by an enlargement, centre  $(0, 2)$  and scale factor  $-2$ .

Draw, and label, triangle  $C$  on the diagram. [2]

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ON16/11/15



- (a) Describe the **single** transformation that maps triangle *A* onto triangle *B*.

*Answer* .....

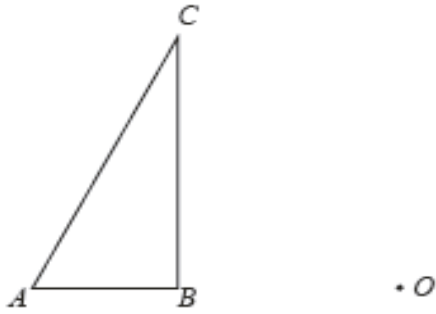
..... [2]

- (b) Triangle *A* is mapped onto triangle *C* by an enlargement, centre  $(0, 2)$  and scale factor  $-2$ .

Draw, and label, triangle *C* on the diagram. [2]

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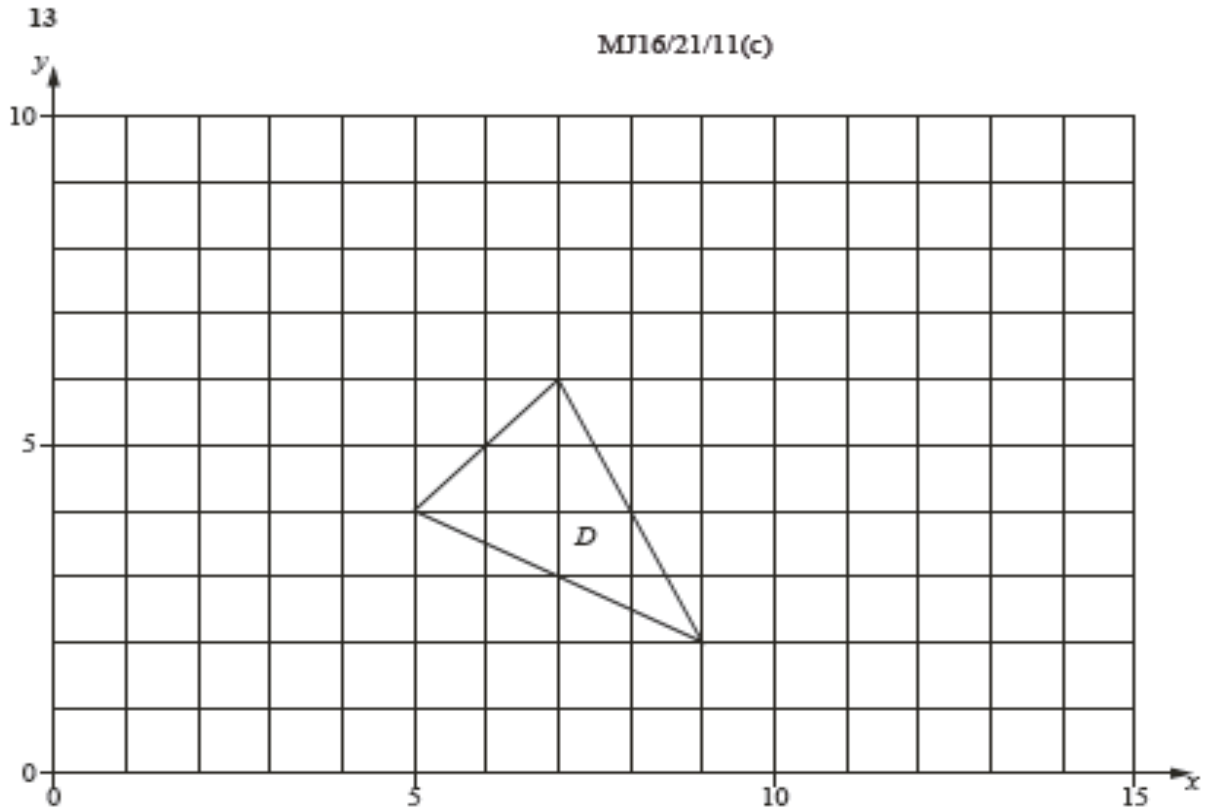
- 12 Triangle  $ABC$  is mapped onto triangle  $A'B'C'$  by a rotation, centre  $O$ , through  $110^\circ$  clockwise.  
Draw and label triangle  $A'B'C'$ . O/N16/12/13



[3]



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The diagram shows triangle  $D$ .

- (i) An enlargement with centre  $(5, 4)$ , scale factor 2, maps triangle  $D$  onto triangle  $E$ .

Draw and label triangle  $E$ .

[2]

- (ii) An enlargement with centre  $(5, 4)$ , scale factor 0.5, maps triangle  $D$  onto triangle  $F$ .

Draw and label triangle  $F$ .

[1]

- (iii) Triangle  $G$  has vertices  $(5, 4)$ ,  $(4, 3)$  and  $(3, 5)$ .

Triangle  $F$  can be mapped onto triangle  $G$  using a single enlargement.

Triangle  $F$  can also be mapped onto triangle  $G$  using a different single transformation  $T$ .

Describe fully the single transformation  $T$ .

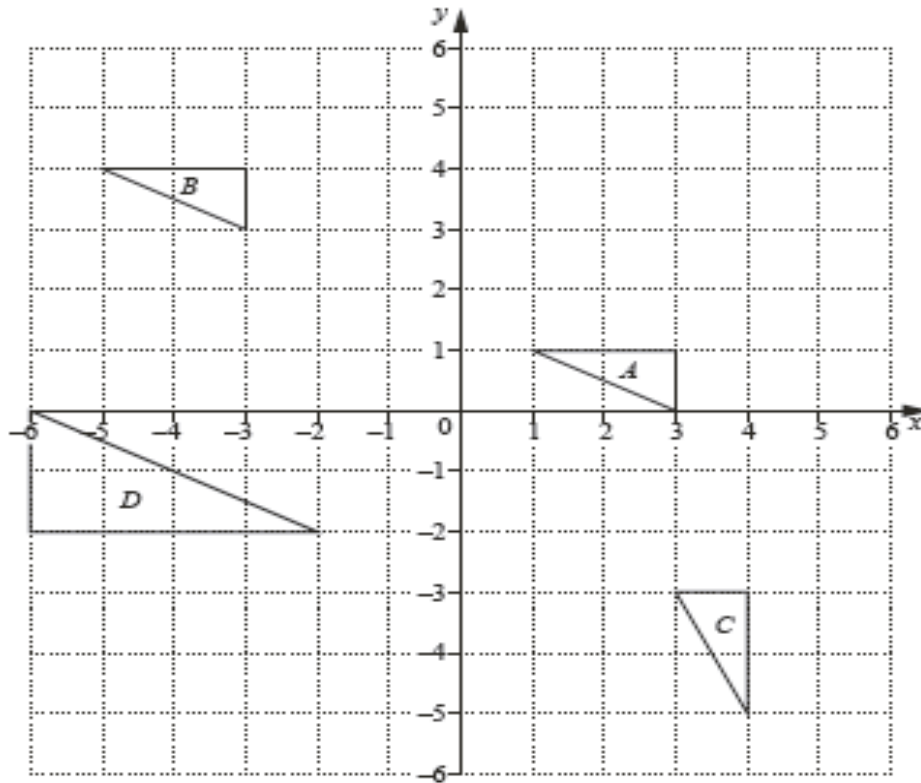
Answer .....

.....

..... [3]

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14 Triangles  $A$ ,  $B$ ,  $C$  and  $D$  are drawn on a centimetre square grid. M/J16/22/10



- (a) The perimeter of triangle  $A$  is  $(a + \sqrt{b})$  cm, where  $a$  and  $b$  are integers.  
Find  $a$  and  $b$ .

*Answer*  $a = \dots\dots\dots b = \dots\dots\dots$  [2]

- (b) Triangle  $A$  is mapped onto triangle  $B$  by the translation  $T$ .  
Write down the column vector that represents  $T$ .

*Answer*  $\begin{pmatrix} \phantom{0} \\ \phantom{0} \end{pmatrix}$  [1]

- (c) Describe fully the single transformation that maps triangle  $B$  onto triangle  $C$ .

*Answer* .....

..... [2]

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(d) Describe fully the single transformation that maps triangle  $B$  onto triangle  $D$ .

*Answer* .....

..... [3]

(e) Write down the matrix that represents the transformation which maps triangle  $D$  onto triangle  $A$ .

*Answer* [1]

(f) The transformation  $V$  is a reflection in the line  $y = 0$ .  
The transformation  $W$  is a rotation  $90^\circ$  clockwise about  $(0, 0)$ .  
The single transformation  $X$  is equivalent to the transformation  $V$  followed by the transformation  $W$ .

(i) The point  $(g, h)$  is mapped onto the point  $P$  by the transformation  $X$ .  
Find the coordinates of  $P$ .

*Answer* (....., .....) [1]

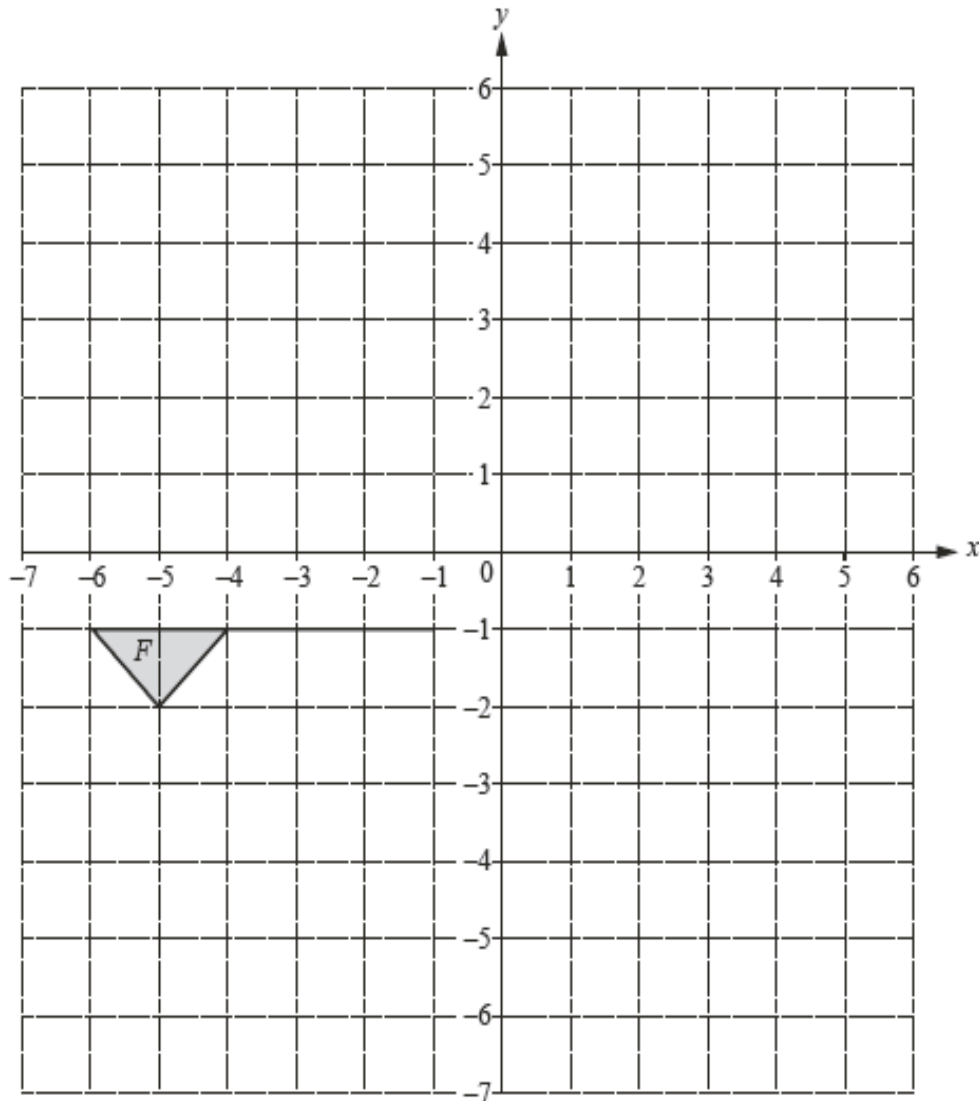
(ii) Describe fully the single transformation  $X$ .

*Answer* .....

..... [2]

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15 (a)

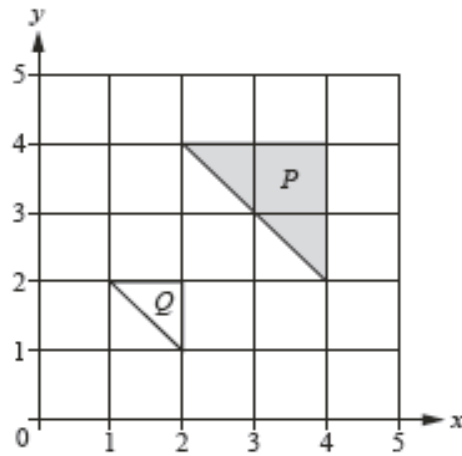


Draw the image of

- (i) flag  $F$  after translation by the vector  $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$ , [2]
- (ii) flag  $F$  after rotation through  $180^\circ$  about  $(-2, 0)$ , [2]
- (iii) flag  $F$  after reflection in the line  $y = x$ . [2]

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(b)



(i) Describe fully the single transformation that maps triangle  $P$  onto triangle  $Q$ .

.....  
 ..... [3]

(ii) Find the matrix that represents this transformation.

(                      ) [2]

(c) The point  $A$  is translated to the point  $B$  by the vector  $\begin{pmatrix} 4u \\ 3u \end{pmatrix}$ .

$|\vec{AB}| = 12.5$

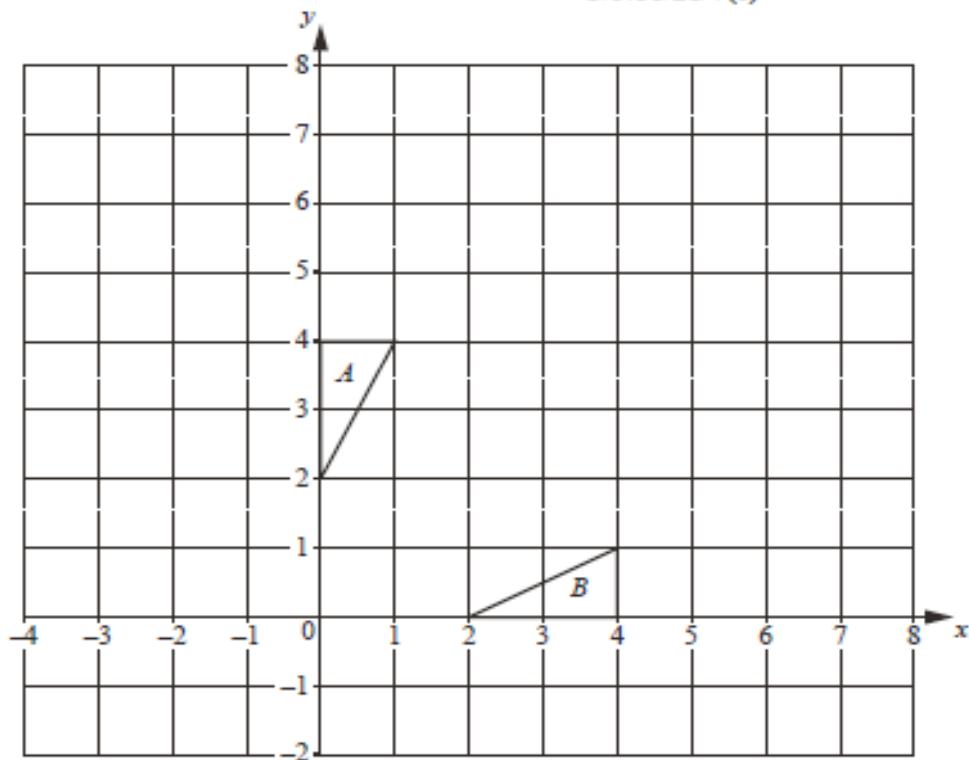
Find  $u$ .

$u = \dots\dots\dots$  [3]

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(16(a))

O/N16/21/7(c)



The diagram shows triangle  $A$  and triangle  $B$ .

- (i) Triangle  $A$  is mapped onto triangle  $C$  by the translation  $P$  with vector  $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ .

Draw and label triangle  $C$ .

[2]

- (ii) Triangle  $A$  is mapped onto triangle  $B$  by a reflection  $Q$ .

Write down the equation of the line of this reflection.

Answer ..... [1]

- (iii) Triangle  $C$  is mapped onto triangle  $D$  by reflection  $Q$ .

Describe fully the single transformation that maps triangle  $B$  onto triangle  $D$ .

Answer ..... [2]

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(iv) Transformation R is a reflection in the line  $y = 0$ .

$$RQ(A) = E.$$

(a) Find the coordinates of the vertices of triangle E.

*Answer* ..... [1]

(b) Describe fully the single transformation that maps triangle A onto triangle E.

*Answer* ..... [2]

(c) Find the matrix which represents the transformation that maps triangle A onto triangle E.

*Answer* ..... [1]

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Marking Scheme

1(a)	Trapezium drawn with vertices $(-3,-1)$ $(-1,-1)$ $(1,-3)$ $(-3,-3)$	<b>1</b>	
1(b)	Trapezium drawn with vertices $(1,1)$ $(1,2)$ $(0,1)$ $(-1,2)$	<b>2</b>	<b>B1</b> for correct size and orientation, wrong centre or for correct enlargement scale factor $\frac{1}{2}$ centre $(1, 0)$ .
1(c)(i)	Reflection in the $x$ -axis or $y = 0$	<b>2</b>	<b>B1</b> for either
2(a)	Translation $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$	<b>2</b>	<b>B1</b> for translation  <b>B1</b> for $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$
2(b)	Correct enlargement, vertices $(2, 5)$ , $(6, 5)$ , $(6, 3)$	<b>2</b>	<b>B1</b> for correct size and orientation, incorrect position  or for enlargement scale factor 2, centre $(0, 3)$
3(a)(i)	Correct triangle	<b>2</b>	<b>B1</b> for two correct vertices or triangle correct size and orientation
3(a)(ii)	Correct triangle	<b>2</b>	<b>B1</b> for two correct vertices or triangle correct size and orientation
3(a)(iii)	Complete description <b>www</b>	<b>3</b>	<b>B1</b> for Rotation <b>B1</b> for either 90 anticlockwise or centre $(0, 3)$
3(b)	$y = x + 5$	<b>2</b>	<b>B1</b> for either $y = x + k$ , $k \neq 5$ or for $y = mx + 5$ , $m \neq 0$ or 1
3(c)	$y = -x$	<b>1</b>	



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4(a)	Reflection and $y = -x$ oe	2	B1 for either
4(b)	Triangle with vertices (1,0), (3,0), (3,1)	2	B1 for 90° clockwise rotation with wrong centre, or for the triangle with vertices (-1,0), (-3,0), (-3,-1)
4(c)	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	1	

5(a)(i)	Triangle B at (-4, -2), (-6, -2), (-4, -6)	2	B1 for two vertices correct or two correct pairs of coordinates soi or correct size and orientation but wrong position
5(a)(ii)	Enlargement, centre (0, 0) oe, scale factor -2	2	B1 for enlargement
5(a)(iii)	1 : 4 oe	1	
5(b)	Triangle C at (-4, 2), (-6, 2), (-4, 6)	2	FT reflection of <i>their</i> triangle B in x-axis BIFT for two vertices correct
5(c)	$\frac{1}{3}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ isw	2	B1 for $k\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ oe with $k \neq \frac{1}{3}$ or for $\frac{1}{3}\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$

6(a)	2	1	
6(b)	Triangle with vertices (5, -1), (8, -1), (8, 1)	2	B1 for two correct vertices, soi or M1 for a line joining (10, -4) to a vertex of triangle B.
6(c)	$\begin{pmatrix} 5 \\ -1 \end{pmatrix}$	1	

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6(a)(i)	Triangle <i>B</i> at (2, -3), (3, -3), (3, -5)	2	<b>B1</b> for translation of correct triangle B
6(a)(ii)	Triangle <i>C</i> at (3, 3), (3, 9), (6, 3)	2	<b>B1</b> for two vertices correct or for $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ oe
6(a)(iii)	$\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ oe	1	
6(a)(iv)	Enlargement Centre (3, -1.5) SF $-\frac{1}{3}$	3	<b>B1</b> for each

7(a)(i)	Triangle <i>B</i> at (2, -3), (3, -3), (3, -5)	2	<b>B1</b> for translation of correct triangle B
7(a)(ii)	Triangle <i>C</i> at (3, 3), (3, 9), (6, 3)	2	<b>B1</b> for two vertices correct or for $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ oe
7(a)(iii)	$\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$ oe	1	
7(a)(iv)	Enlargement Centre (3, -1.5) SF $-\frac{1}{3}$	3	<b>B1</b> for each

8(a)(i)	Triangle <i>B</i> at (4, -1), (4, -4), (5, -4)	2	<b>B1</b> For triangle B the correct size and orientation
8(a)(ii)	Triangle <i>C</i> at (1, 4), (3, 4) (3, -2)	2	<b>B1</b> for correct size and orientation, incorrect position <b>or</b> for triangle with two vertices correct <b>or</b> for triangle at (-3, 0), (-5, 0), (-5, 6)

9(a)	<b>B</b> drawn with vertices (2,-3) (3,-3) (3,-5)	1	
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10(a)	Reflection $y = -x$ oe	2	C1 for reflection or for $y = -x$ oe
10(b)	Triangle vertices $(-1, 2)$ , $(-1, 5)$ , $(-2, 4)$	2	C1 for correct size and orientation, incorrect position or for $90^\circ$ clockwise rotation about origin

11	(a) Rotation $90^\circ$ clockwise oe, centre $(3, 1)$	1 1	Mark lost if a second transformation is named.  B1 for two correct vertices, or for vertices $(2, 0)$ , $(4, 0)$ , $(4, 4)$
	(b) vertices: $(-2, 4)$ , $(-4, 0)$ , $(-4, 4)$	2 *	

12	Correct triangle	3*	Following an attempt at a rotation of $110^\circ$ about $O$ , award C2 for two correct vertices or C1 for one correct vertex.  If [0] scored then either B1 for arc(s) of correct radii, centre $O$ , (from $A$ , $B$ or $C$ ); or B1 for $AOA'$ or $BOB'$ or $COC' = 110^\circ$
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13	(i) Triangle vertices $(5,4)$ , $(13,0)$ , $(9,8)$	2	B1 for 2 correct   B2 for Rotation with either centre or angle.  B1 for Rotation.
	(ii) Triangle $F$ $(5,4)$ , $(7,3)$ , $(6,5)$	1	
	(iii) Rotation $180^\circ$ Centre $(5,4)$	3	

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14	(a)	a = 3 b = 5	2	B1 for one correct
	(b)	$\begin{pmatrix} -6 \\ 3 \end{pmatrix}$ or $3 \begin{pmatrix} -2 \\ 1 \end{pmatrix}$	1	
	(c)	Reflection, $y = x$	2	B1 for reflection or B1 for $y = x$ only
	(d)	Enlargement, Scale factor - 2, centre (-4, 2)	3	B1 for enlargement / negative enlargement B1 for scale factor - 2 B1 for centre (-4, 2)
	(e)	$\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$ oe	1	

	(f) (i)	(-h, -g)	1	
	(ii)	Reflection $y = -x$	2	B1 for reflection or B1 for $y = -x$ only

15(a)(i)	Correct translation	2	B1 for translation $\begin{pmatrix} 6 \\ k \end{pmatrix}$ or $\begin{pmatrix} k \\ -2 \end{pmatrix}$
15(a)(ii)	Correct rotation	2	B1 for rotation $180^\circ$ but other centre
15(a)(iii)	Correct reflection	2	B1 for reflection in $y = -x$
15(b)(i)	Enlargement [factor] $\frac{1}{2}$ or 0.5 [centre] (0, 0) oe	3	B1 for each
15(b)(ii)	$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ oe	2	B1 for matrix of form $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ oe, $k \neq 0$ or 1

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16 (i)	$C$ at (3, 1) (3, 3) (4, 3)	2	<b>B1</b> for either vertical or horizontal correct Or for two vertices correct and correct orientation
(ii)	$y = x$ oe	1	
(iii)	Translation $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$	2	<b>B1</b> for translation or $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ Or <b>M1</b> for $D$ seen at (1, 3), (3, 3), (3, 4)
(iv) (a)	(2, 0) (4, 0) (4, -1)	1	
(b)	Rotation, $90^\circ$ clockwise, (0,0) oe	2	<b>B1</b> for two correct from: Rotation, $90^\circ$ clockwise oe, (0, 0) oe
(c)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	1	